CHURCHLANDS SENIOR HIGH SCHOOL
MATHEMATICS SPECIALIST 3, 4 TEST TWO 2017
Non Calculator
Chapters 3, 4,

Name________________________________

Time: 40minutes
Total: 40 marks

1. [8 marks: 1,1,2,2,2]
Given that $f(x) = \frac{1}{x+2}$ and $g(x) = x - 5$

a) State the natural domain of g.

$D_x = \{x : x \in \mathbb{R}\}$ ✓

b) Explain clearly why the domain for g has to be restricted if fog is to be a function.

Domain needs to be restricted because f cannot take on the value $x = -2$ ✓

$\therefore$ $x = 3$ needs to be eliminated from D of g to stop that happening

c) State the largest possible domain for fog and the corresponding range.

$D_{fog} = \{x : x \in \mathbb{R}, x \neq 3\}$ ✓

Domain $\mathbb{R}$ $\xrightarrow{g}$ Range $\mathbb{R}$

Domain $\{\mathbb{R}, x \neq -2\}$ $\xrightarrow{f}$ Range $\{\mathbb{R}, y \neq 0$

$R_{fog} = \{y : y \in \mathbb{R}, y \neq 0\}$ ✓

d) Evaluate $gof(-\frac{5}{2})$.

$g\left[f\left(-\frac{5}{2}\right)\right] = g\left[\frac{1}{-\frac{1}{2}}\right] = g[-2] = -7$ ✓ ✓

e) Express in simplest form $fof(x)$.

$f\left(\frac{1}{x+2}\right) = \frac{1}{\frac{1}{x+2} + 2} = \frac{1}{\frac{1}{x+2} + \frac{2(x+2)}{x+2}}$ ✓

$= \frac{1}{\frac{2x+5}{x+2}}$

$= \frac{x+2}{2x+5}$ ✓

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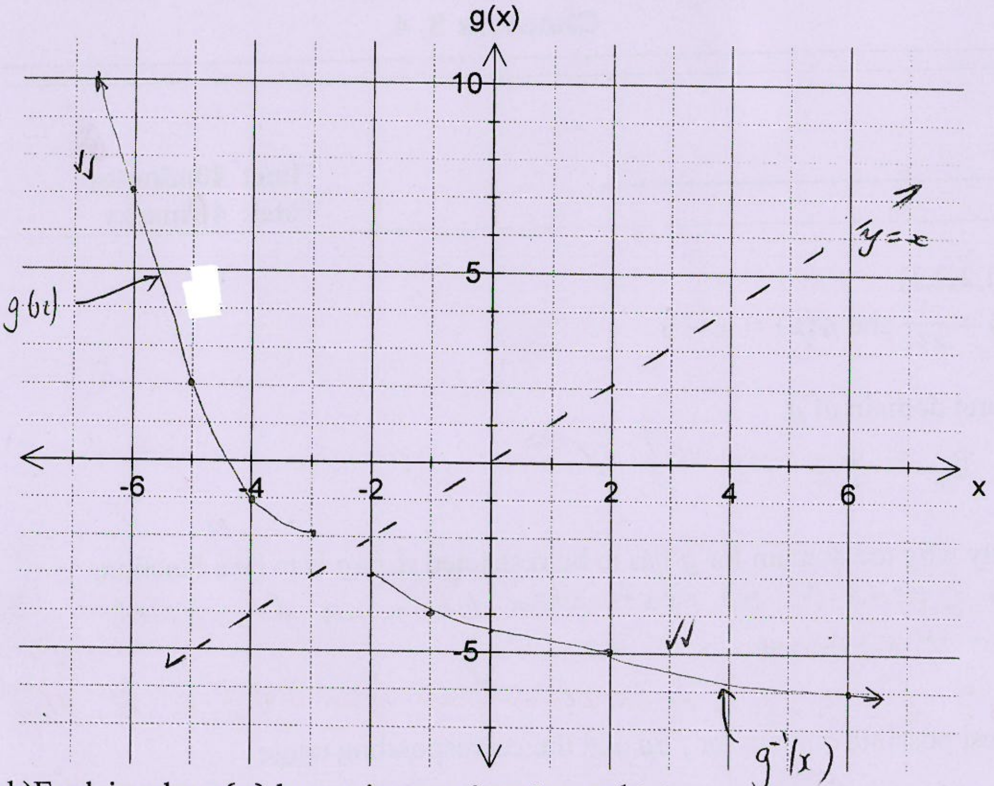
2.[11 marks:2,1,3,2,1,2]

$g(x) = x^2 + 6x + 7 \ for \ x \in (-\infty, -3]$.

a) Sketch the graph of g on the axes provided.



b)Explain why $g(x)$ has an inverse function $g^{-1}(x)$.

g(x) has an inverse because it is one-to-one ✓

c) Find algebraically, a formula for $g^{-1}(x)$.

$y = x^2 + 6x + 7$

interchange x & y

$x = y^2 + 6y + 7$

$x = y^2 + 6y + 9 - 9 + 7$

$x = (y+3)^2 - 2$ ✓

$x + 2 = (y+3)^2$

$\pm\sqrt{x+2} = y + 3$

$-3 \pm \sqrt{x+2} = y$ ✓

require only $y = -3 - \sqrt{x+2}$ ✓

$\therefore g^{-1}(x) = -3 - \sqrt{x+2}$

d) Sketch the graph of $g^{-1}(x)$ on the same axes as $g(x)$ above.

e) Find the range of $g(x)$.

Range of $g(x) = \{y : y \geq -2\}$ ✓

f) Find the domain and range of $g^{-1}(x)$.

Domain of $g^{-1}(x) = \{x : x \geq -2\}$ ✓

Range of $g^{-1}(x) = \{y : y \leq -3\}$ ✓

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3.[12 marks:2,~~2~~1,2,2,~~4~~5]

Consider the curve with equation $y = \frac{x^2-9}{x^2+x-6}$

a) State the equation of all asymptotes.

$y = \frac{(x-3)(x+3)}{(x-2)(x+3)}$ ✓

$= \frac{x-3}{x-2}$, $x \neq -3$

$= \frac{x-2-1}{x-2} = 1 - \frac{1}{x-2}$

∴ Vertical asymptote at $x = 2$

Horizontal asymptote at $y = 1$. ✓

b) Identify the point of discontinuity on this curve.

point of discontinuity at $x = -3$. ✓

$\left(-3, \frac{6}{5}\right)$

c) State the x and y intercepts

x intercept $\Rightarrow y = 0$

ie at $(3, 0)$ ✓

y-intercept $\Rightarrow x = 0$

ie at $\left(0, \frac{-9}{-6}\right)$

ie at $\left(0, +\frac{3}{2}\right)$ ✓

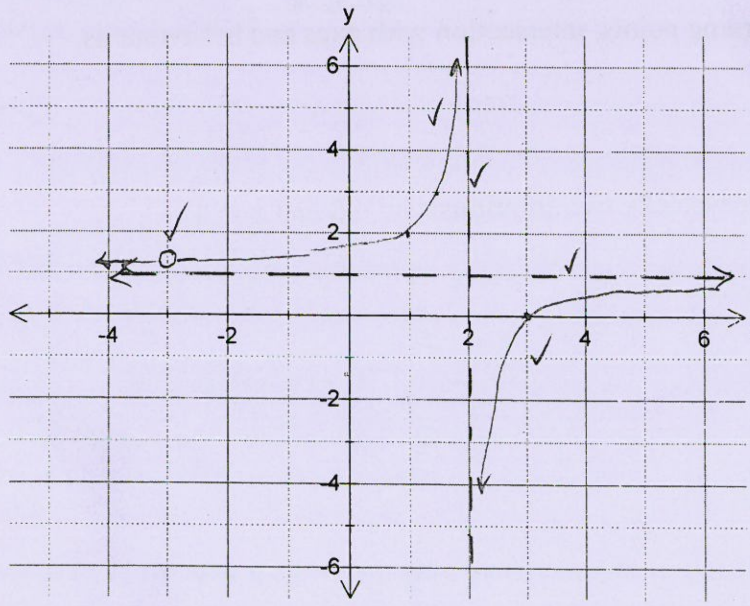
d) i) State the limit as $x \to +\infty$

ii) State the limit as $x \to -\infty$

as $x \to +\infty$, $y \to 1^-$ ✓

as $x \to -\infty$ $y \to 1^+$ ✓

e) Sketch the curve on the axes provided highlighting all the main features clearly.



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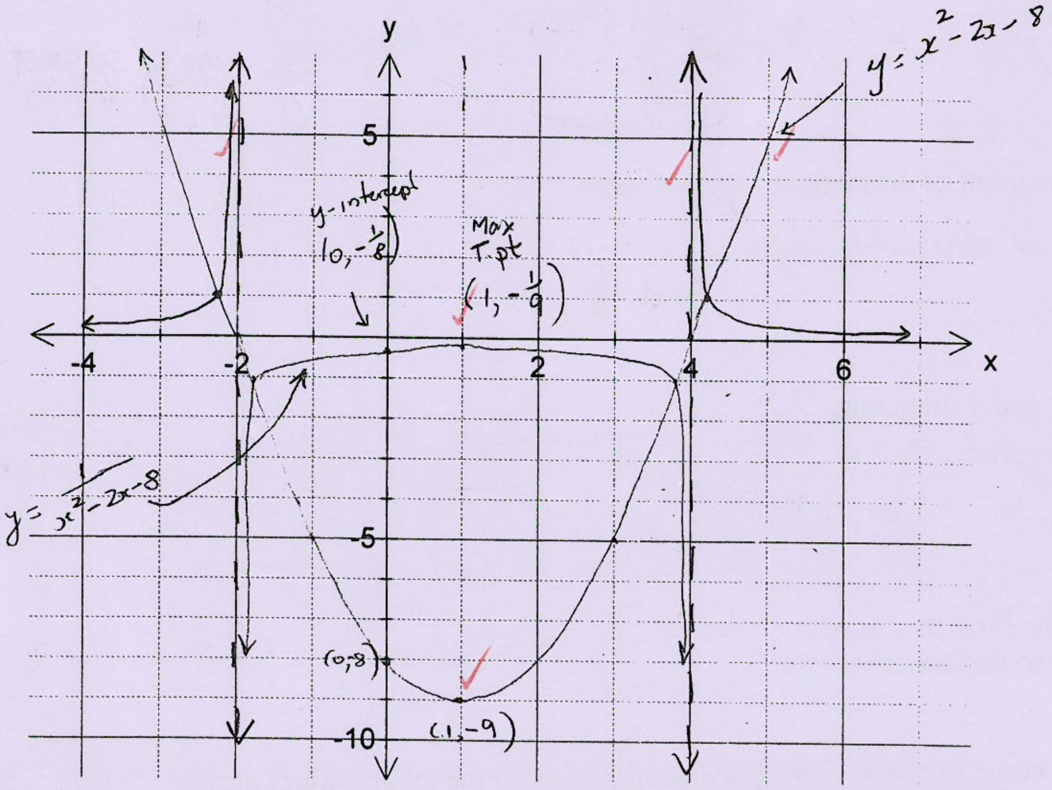
6.[~~7~~ 6 marks:3,3]

a) On the axes provided neatly sketch the graph of $y = x^2 - 2x - 8$.
Clearly indicate the i) x intercepts ✓
ii) the $y$intercept ✓
iii) the coordinates of the turning point. ✓



b) Use your previous graph to help you draw the graph of $y = \frac{1}{x^2-2x-8}$. on the same set of axes.
Clearly indicate any asymptotes, turning points, intersection with axes and behaviour as $x \to \pm\infty$.

5. [3 marks]
The equation $|x-4| = |2x+k|$ has exactly two solutions $x = -5$ and $x = 1$.
Find the value(s) of k.

Two possibilities $x - 4 = 2x + k$ or $x - 4 = -(2x + k)$

$\Rightarrow$ for $x = -5$
$-5 - 4 = 2(-5) + k$
$-9 = -10 + k$
$\therefore k = 1$

for $x = -5$
$-5 - 4 = -(-10 + k)$
$-9 = 10 - k$
$k = 19$

$\Rightarrow$ for $x = 1$
$1 - 4 = 2(1) + k$
$-3 = 2 + k$ ✓
$k = -5$

for $x = 1$
$1 - 4 = -(2 + k)$
$-3 = -2 - k$ ✓
$k = 1$

Hence in order to make both cases true,
$k = 1$ is the only solution ✓

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